A NEW EXPERIMENTAL LIMIT ON THE VALIDITY OF LOCAL POSITION INVARIANCE

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ABSTRACT

The validity of Local Position Invariance (LPI) was tested by comparing two different kinds of atomic frequency standards in a time-varying gravitational potential $U(t)$, the variation being due to the earth's annual elliptical orbital motion. $U(t)/c^2$ varies between plus and minus $3.3 \cdot 10^{-10}$ during one year (c: speed of light). Comparing a caesium atomic fountain frequency standard with a hydrogen maser during about one year allowed to set a new upper limit on the variation of the frequency difference synchronous with the variation of $U(t)$. The limit is now $2.1 \cdot 10^{-5}$ of the above mentioned amount, reduced by a factor of more than 30 compared to the result of a previous experiment.

Key Words: Atomic Clocks, General Relativity, Local Position Invariance

1. INTRODUCTION

Time and frequency metrology is one of the rare fields in physics where general and special relativity manifest themselves immediately and need to be taken into account in the everyday practise of comparing clock rates and time scales over long distances. Therefore the methods of time metrology are also well suited for experimental tests of theories of relativity or for the search of variations of the fundamental constants. Such tests have recently gained renewed interest [Refs. 1-4], stimulated by the availability of frequency standards with improved characteristics compared to the situation in previous years. The analysis described in this paper [Ref. 7] resumes earlier work [Refs. 5,6], searching for a time variation of frequency differences between non-identical atomic clocks subjected to the same time variations of the local gravity potential. The theoretical background is summarised in the following section. Section 3 contains a description of the experimental procedures. The paper concludes with a discussion of the results and an outlook at future trends in this field.

2. THEORETICAL BACKGROUND

The principle of equivalence has played an important role in the development of gravitation theory. One part of Einstein's Equivalence Principle (EEP) which is a basic element of General Relativity [Ref. 8] is known as Local Position Invariance (LPI), stating that "the outcome of any local non-gravitational test experiment is independent of where and when in the universe it is performed" [Ref. 9]. The gravitational redshift of the clock frequency,

$$y_U = (\nu_U - \nu_0) / \nu_0 = \Delta U/c^2 , \qquad (1)$$

is a consequence of EEP [Ref. 8]. The frequency difference $\nu_U - \nu_0$ occurs because of the difference ΔU in Newtonian gravitational potential between the clock's actual location (in space or time) and a reference value for which the clock frequency is ν_0. If LPI was not fulfilled, just to mention one possible consequence, equation (1) would have to be modified by

$$y_U = (1+\beta_k) \cdot \Delta U/c^2 . \qquad (2)$$

The parameter β_k would be a function of either the position or of the atomic species k used, or of both. The most refined experiment made in order to test the condition $\beta_k = 0$ was the "Gravity Probe A" mission during which the frequency of a hydrogen maser (H) on board of a Scout rocket was recorded with reference to a stationary ground based maser. A limit of $|\beta_H| < 7 \cdot 10^{-5}$ was deduced [Ref. 10]. Another type of experiment made in this context comprised the comparison of stationary frequency standards based on two different atomic species (a and b) subjected to the same variation of the gravitational potential. The relative frequency differences

$$y_a - y_b = (\beta_a - \beta_b) \cdot U(t)/c^2 \qquad (3)$$

were determined, varying synchronously with the diurnal variation of the local gravity potential due to earth rotation [Ref. 5] or the annual variation due to the eccentricity of the earth orbit around the sun [Ref. 6]. We report here on an experiment of the latter kind. The earth's orbital motion entails a temporal variation of the solar gravitational potential on earth, described by

$$U(t)/c^2 = - 2\, G\, M_S / (a \cdot c^2)\, e \cos(\varphi(t)) , \qquad (4)$$

where the product of the gravitational constant G and the solar mass M_S amounts to $1.327 \cdot 10^{20} \mathrm{m}^3/\mathrm{s}^2$, $a = 1.496 \cdot 10^{11}$ m is the semimajor axis of the earth orbit, and $e = 0.0167$ is the eccentricity of the earth orbit. $\varphi(t)$ is the true anomaly, zero at perihelion which occurs early in the year, e. g. on Jan 4th 2001 (Modified Julian Date MJD 51913). The peak to peak variation in $U(t)/c^2$ thus amounts to $0.66 \cdot 10^{-9}$. Of course, this variation is undetectable using clocks which experience the same $U(t)$ as long as LPI is fulfilled.

The difference $(\beta_a - \beta_b)$ would become non-zero if the (non-gravitational) fundamental constants which determine the energy of hyperfine states, e. g. the fine structure constant α, would be a function of the external gravitational potential [Refs. 5,6]. In fact, the hyperfine splittings in the ground state of hydrogen and caesium, the two atomic species of interest in the context of this study, are known to have a different dependence on α [Ref. 11]. The temporal variation of the hyperfine splitting frequencies of caesium and hydrogen simultaneously recorded with the variations of the gravity potential allows to deduce a limit on $|\beta_a - \beta_b|$, but of course not on the individual terms β_a or β_b.

3. DESCRIPTION OF THE EXPERIMENT AND THE RESULTS

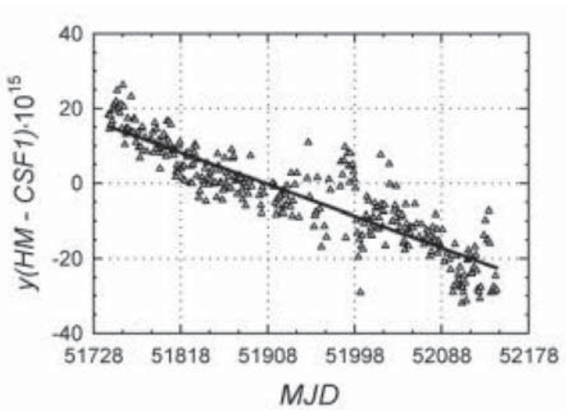

Figure 1. *Results of comparisons of a hydrogen maser HM with respect to CSF1, PTB's caesium atomic fountain frequency standard, expressed as relative frequency difference y: =* y_H *-* y_{Cs} *as a function of time. Here and in all other figures MJD designates Modified Julian Date. MJD 51726 corresponds to 2000-07-01. The solid line represents a least-squares fit to the data points and is explained further in the text.*

Almost continuous records of the frequency of an active hydrogen maser with reference to the SI Hertz, as realised with PTB's atomic fountain frequency standard CSF1, are available since summer 2000. Operation and uncertainty evaluation of CSF1 have been described elsewhere [Refs. 12, 13]. The hydrogen maser is operated including a cavity tuning procedure whereby the resonance frequency of the maser's microwave cavity is tuned to the hydrogen resonance frequency. The tuning requires a stable reference signal delivered by a second maser [Ref. 14]. The remaining maser frequency drift is not calculable from first principles. It is said to be due to the ageing of the Teflon coating covering the inner surface of the storage bulb [Ref. 15]. The raw comparison data are displayed in Fig. 1. Each data point (total number 321) represents an average over a measurement interval between 16 hours and 24 hours. The relative statistical measurement uncertainty for each point varies between $1\cdot10^{-15}$ and $3\cdot10^{-15}$. The excess noise during the middle period reflects an imperfect tuning of the maser cavity caused by the lack of the required stable reference frequency.

From a linear least-squares fit, the drift of the maser frequency was deduced as - $0.094\cdot10^{-15}$ / day and was determined with a standard uncertainty of $0{,}003\cdot10^{-15}$ / day (1 σ). Maser frequency drifts of similar small values and with both signs were also reported elsewhere [Refs. 14,16].

Removal of the constant drift from the data in Fig. 1 results in de-trended data, depicted in Fig. 2. Least-squares fits using equal statistical weight for each

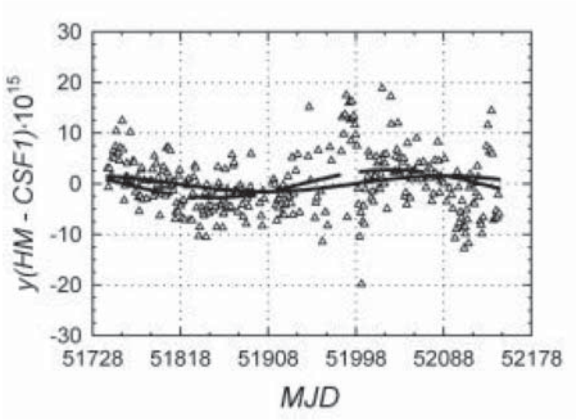

Figure 2. *De-trended data from Fig. 1. The two sinusoidal curves represent least-squares fits with an annual period using equal statistical weight for each data point. The broken line represents a simultaneous fit of amplitude and phase whereas the solid line represents a fit with the phase* Φ *fixed so that a maximum (or minimum) resulted at perihelion (MJD 51913).*

data point were made, giving two sinusoidal curves, $y_G \cdot \sin(2\pi\, t / 365 + \Phi)$, t expressed in days, with an annual period. The broken line represents a simultaneous fit of amplitude and phase which yields $y_G = 2.76(44)\cdot10^{-15}$ and Φ corresponding to a maximum at MJD = 51861(9). Here, and in the following, values in brackets represent 1σ-standard uncertainties. For the fit represented by the solid line, Φ was fixed so that a maximum (or minimum) resulted at perihelion. Under this constraint $y_G = 1.32(45)\cdot10^{-15}$ was found. When the data set is split into two halves, taking always one point out of two, two independent fits to the two data sets give almost the same y_G values with uncertainties increased by about a square root of two, as expected.

When a linear and a sinusoidal term is fitted in one step to the original data of Fig. 1, the amplitude of the sinusoid comes out somewhat larger than said before. A maximum of $y_G = 4.25(54)\cdot10^{-15}$ was obtained in case that no constraint on the phase of the sinusoid was imposed. Presumably, the process of de-trending absorbs partially the Fourier component with a frequency 1/year from the original data which only span about one year. For the estimate of $|\beta_a - \beta_b|$ we will later use the last y_G value which is the largest observed. But in the following discussion we show always de-trended data as they allow a better interpretation of the results.

4. DISCUSSION

As an outcome of a previous study [Ref. 6], making use of the comparison of a magnesium based frequency standard at 601 GHz and a commercial caesium clock, the equivalent fitted amplitude was stated as about 10^{-13} in magnitude. The benefit of using more stable frequency standards and collecting a larger number of data is obvious. However, it appears not straightforward to safely state an upper bound for $|\beta_a - \beta_b|$. The observation of an annual frequency variation with its maximum shifted in time with respect to perihelion calls for an examination of other possible frequency shifting effects in CSF1 as well as in the hydrogen maser. It appears very probable that the apparent statistically significant frequency variation synchronous with the variation of $U(t)$ (solid curve in Fig. 2) has a more trivial "technical" cause.

At first we consider CSF1. Its uncertainty was evaluated for the first time in early 2000 [Ref. 12]. Since February 2001 (Modified Julian Date MJD > 51950) CSF1 has been operated including a selection of atoms in one hyperfine sub-state prior to the excitation of the clock transition. Thereby the CSF1 uncertainty could be reduced to $1.0 \cdot 10^{-15}$ for a certain standard operation condition [Ref. 17]. During the period under study, CSF1 was also operated at conditions which deviated therefrom, and we estimate the "average" CSF1 uncertainty for the whole period to $2.0 \cdot 10^{-15}$.

An uncertainty estimate reflects the knowledge of the effects which might systematically shift the otherwise unperturbed clock transition frequency. It also implies that the CSF1 frequency should not exhibit fluctuations which exceed plus or minus twice the stated uncertainty during more than 5% of the operating time. This latter statement, of course, neglects the short-term frequency instability. In case of CSF1 it is characterised by $\sigma_y(\tau) \leq 3 \cdot 10^{-13}/(\tau/\mathrm{s})^{1/2}$ and calls for sufficiently long measurement times. For averaging times τ exceeding half a day the observed frequency variations (see Figs. 1 and 2) are to a large extent determined by the available hydrogen maser. Ideally, any long-term fluctuations of CSF1 should be significantly smaller than the stated uncertainty, but due to the lack of a frequency reference, similar in quality to CSF1, this is difficult to verify. Although it appears very improbable that the observed (annual) frequency variations were caused by CSF1 we admit an additional factor in the estimate of $|\beta_a - \beta_b|$ which allows for the CSF1 uncertainty.

It is a challenging task to achieve the required long-term stability and, in particular, the sufficiently low temperature sensitivity of the frequency of a hydrogen maser. Without cavity tuning, the cavity temperature would have to be stabilised to better than about 20 µK for to achieve a frequency instability of below $1 \cdot 10^{-14}$, as calculated from table 6.7.3 in [Ref. 15]. The manufacturer of PTB's hydrogen maser specified a potential frequency dependence on ambient temperature of below $\pm 10^{-14}$/K. This specification was later said to be conservative as, in case of a proper function of the cavity auto-tuning system and of the two-stage temperature control shields around the maser cavity, the relative frequency dependence should be only about $2 \cdot 10^{-15}$ /K [Ref. 18]. The hydrogen maser and CSF1 were operated in the same temperature controlled room whose air temperature is constantly monitored. In Fig. 3 the recorded temperature values and a sinusoidal least squares fit with an annual period are depicted (right scale), together with the fit curves to the frequency data as in Fig. 2. The temperature variations exhibit an annual term with a peak-to-peak variation of 0.2 K and a minimum around the end of February, which is close to the occurrence of the lowest temperatures in winter 2001 in Braunschweig.

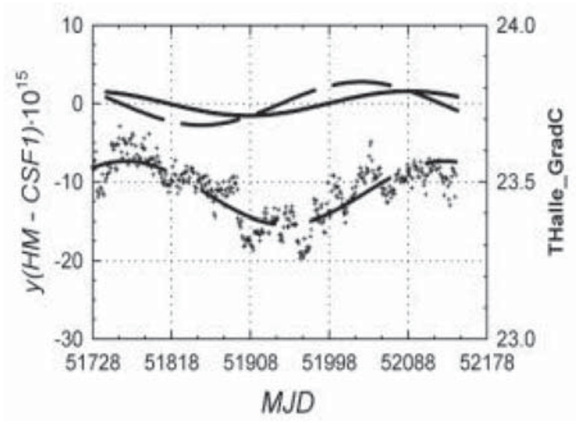

Figure 3. Recorded air temperature in PTB's clock room (right scale) and least-squares fit of a sinusoid with an annual period shown as dashed-dotted curve. For comparison, the two fit curves from Fig. 2 have been included (left scale).

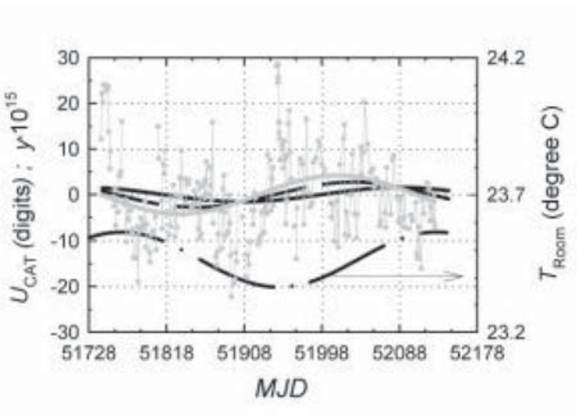

Figure 4. Cavity control voltage U_{CAT} of the maser's cavity tuning system (left scale), expressed in digits of the digital-to-analogue converter. A linear trend of -1.9 digit/day was subtracted and a least-squares fit of a sinusoid with an annual period is shown as grey curve. As one digit approximately corresponds to a step of $1 \cdot 10^{-15}$ in relative maser frequency it is possible to plot the sinusoid fit curves from Fig. 2 in the same scale. For comparison, the temperature fit from Fig. 3 is added (right scale).

The control voltage fed to the maser's cavity tuning varactor is generated from a 16-bit digital to analogue converter (DAC) whose state is displayed. Records of the DAC values show indeed a small annual term, superimposed to the dominant linear trend which has been removed for display of the DAC readings in Fig. 4. Assuming that the linear trend reflects the ageing of the cavity structure, one would expect a signature super-imposed which reflects the variations of the temperature of the cavity. The observed annual term, however, is almost out of phase by $\pi/2$ with respect to the temperature variations so as if it was proportional to their time derivative. No explanation can be given for that finding so far. As one step of the least-significant bit of the DAC reading approximately corresponds to a step of $1{\cdot}10^{-15}$ in relative maser frequency [Ref. 18] the observed annual term could well explain the observed annual frequency variations out of phase with the variation of the gravitational potential. All four fitted sinusoids are overlaid in Fig. 4 as a proof.

As long as one cannot exclude that frequency variations due to technical reasons might mask or mimic the effect under study one has to estimate the potential magnitude $|\beta_a - \beta_b|$ with some care. Our current estimate (67% confidence level) is

$$|\beta_a - \beta_b|$$

$$\leq (4.3 + 0.5 + 2){\cdot}10^{-15} / 0.33{\cdot}10^{-9}$$

$$= 2.1{\cdot}10^{-5}.$$

Here the first term represents the maximum observed amplitude of an annual variation, the second term represents the 1σ- standard uncertainty of the particular fit, and the third term is the average CSF1 1σ- combined uncertainty during the measurement period. The new limit on $|\beta_a - \beta_b|$ is about a factor of 30 tighter than the previously verified value [Ref. 6]. If one would consider strictly only the sinusoidal variation in phase with the variation of the gravity potential (solid line in Fig. 2), the limit on $|\beta_a - \beta_b|$ would even be tighter by another factor of 2.

5. CONCLUSION

With the development of more stable and more accurate frequency standards than available in previous years, new and improved experimental tests of gravitation theories have become feasible. In addition to ground based tests as the one reported here, several space projects were proposed [Ref. 19] or were already approved [Ref. 20] during which such tests shall be conducted in space environment.

On the other hand, we have to concede that such analyses are currently more "en vogue" than in the past, and that in the past it had been overlooked to analyse data in this context. As a proof we recall data comparing PTB's primary clock CS2 [Ref. 13] and an active hydrogen maser in the years 1991 - 1993 which had been published in another context before [Ref. 14]. Now, we treat the data in the same way as explained in section 2, and we get a very similar result as from the analysis of the CSF1 data, the results being depicted in Fig. 5. The maximum amplitude of a sinusoid synchronous with the annual variation of the gravity potential comes out as $1.15(1.28){\cdot}10^{-15}$ whereas a sinusoid almost in phase with the recorded temperature variations in PTB's clock hall in those days has an amplitude of $2.415(1.27){\cdot}10^{-15}$. One has to know that the CS2 type B uncertainty as well as its short-term frequency instability is larger than that of CSF1 by a factor of 10 [Ref. 13]. In addition, not all maser operational parameter data are still known now. So we refrain here from stating a limit on the validity of LPI based on these old data. However, they represent a strong evidence that the current result is trustworthy.

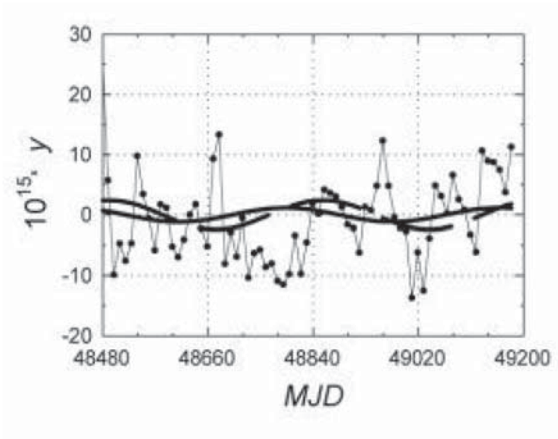

Figure 5. *Results of comparisons of a hydrogen maser with respect to CS2, PTB's primary clock, expressed as relative frequency difference, y: = y_H - y_{Cs}, as a function of time. Data were taken between 1991 and 1993. Each point represents a ten-day average value, and a linear drift of $-1.58{\cdot}10^{-16}$/day was removed from the data. MJD 48480 corresponds to 1991-08-12. The broken line represents a simultaneous fit of amplitude and phase whereas the solid line represents a fit with Φ fixed so that a maximum (or minimum) resulted at perihelion.*

The roads for further improvements are well prepared. CSF1 can now be operated quasi continuously, and it is obvious from the discussion above that comparison with one or two well-behaving hydrogen masers over periods of a few years will allow to confirm the validity of LPI much tighter than said in this contribution.

At PTB we may in the near future start regular comparisons between an optical frequency standard based on a single trapped ytterbium ion [Ref. 21] and CSF1. The recent development of comparatively easy means for measurements of optical frequencies [Ref. 22] facilitates such studies. It was estimated that the ytterbium frequency standard currently realises the unperturbed transition frequency of the 6s $^2S_{1/2}$ (F=0) - 5d $^2D_{3/2}$ (F=2) transition in $^{171}Yb^+$ at λ = 435 nm with a relative uncertainty below 10^{-14} [Ref. 23]. Significant improvements can be expected in the oncoming years. Such optical frequency measurements could also be used for testing the validity of LPI much tighter.

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